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Characterization of Inductive loop coupling in a Cyclotron Dee Structure

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Abstract

Many of today's low to medium-energy cyclotrons apply RF power to the resonator structure (the dees) by inductive loop coupling through a feed-line driven by an RF transmitter employing a triode or tetrode power tube. The transmitter's output network transforms the tube's optimum load line (typically a few thousand ohms) down to Z_0, typically 50 ohms. But the load-line is not a physical resistance, so one would not expect to see 50 ohms when looking back toward the transmitter. Moreover, if both the resonator's input and the transmitter's output are matched to Z_0, then the coupled or working Q of the resonator is reduced to half that of the uncoupled Q, implying that half the power is being dissipated in the transmitter's output resistance- an inefficient and expensive solution for a high power RF application. More power is available if the transmitter's reverse-impedance is not matched to Z_0, but this may result in misalignment between the frequency for correct forward match at the loop, versus the frequency for maximum power in the resonator. The misalignment can be eliminated, and the working Q maximized, by choosing the appropriate length of feed-line between the non-matched transmitter output and the matched resonator's input. In addition, the transmitter's output impedance may be complex, comprising resistance plus reactance, requiring a further process and means of measuring the output impedance so that an additional compensating length of feed-line can be incorporated. But a wrong choice of overall feed-line length- even though correctly load-matched at the resonator's operating frequency- can result in a curious degenerate condition, where the resonator's working Q appears to collapse, and the potential for transmitter overload increases substantially: a condition to be avoided!

Keywords: Impedance matching; RF power transfer; Q-Circle; Smith Chart, wide-band, degenerate condition

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1.0 Introduction

We'll first present a general formula for z_{in}, the normalized impedance looking into the loop, which will facilitate exploration of the relevant parameter space without regard to specific component values or frequency range. We'll find that the normalized reactance of the isolated coupling loop x_{L1} is of singular importance in the characterization and understanding of overall RF System properties and performance. Setting $z_{in} = 1.0$, i.e., forcing a matched condition, and solving for the real and imaginary parts of the resulting equation, reveals useful inter-relationships between resonator parameters. A further transformation, mapping the z-plane onto the unit circle, yields the complex reflection coefficient, Γ, resulting in the familiar *Smith Chart* representation by which we can plot Γ over a wide enough frequency range to produce a geometric construct called a Q circle, which greatly facilitates visualization of the effects of varying parameters such as resonator Q, coupling and loop reactance, etc..

Transporting the Q circle upstream, away from the load and toward the transmitter through an appropriately chosen length of transmission feed-line allows us to align the axis of the Q circle with the real (resistance) axis of the Smith Chart, thereby revealing a method and formula for estimating the complex output impedance of the transmitter which, in turn, guides the choice of overall resonator-plus-generator feed-line length required to align the frequency for best z_0 match with that for maximum power in the resonator. RF simulation software [Harriman, 2015], combined with 'hands-on' measurements on a High-Q model resonator using a Vector Network Analyzer [SDR kits 2017] provides validation for the results presented.

2.0 Resonator Input Impedance

Near resonance, the cyclotron Dee structure can be modelled as an equivalent lumped circuit. From standard circuit analysis:

$$Z_{in} = V_{in} / I_{in} = j\omega L_1 + \{\omega^2 M^2\} / \{j[\omega L_2 - 1/\omega C] + R\} \quad (1)$$

Adroit manipulation of terms and changes of variable yield a more compact and general formula which is independent of any specific frequency range or specific component values (see Appendix A for derivation):

$$z_{in} = jx_{L1} + (k^2 Q_0\, x_{L1}) / (1 + j\psi) \quad (2)$$

where $\mathbf{z_{in}}$ (lower case) is the normalized impedance with respect to Z_0 (50 ohms) measured directly at the loop's input terminal; $\mathbf{x_{L1}}$ is the normalized self-reactance of the loop– assumed constant near resonance; **k** is the inductive coupling coefficient between loop L_1, and dee stem L_2; $\mathbf{Q_0}$ is the un-coupled Q of the resonator = ω_0 / (bandwidth); $\boldsymbol{\psi} = (\omega-\omega_0) / (\omega_0 / 2Q_0)$ is a generalized frequency variable expressed as a number of half- bandwidths displaced from ω_0, the natural resonant frequency of the un-coupled resonator.

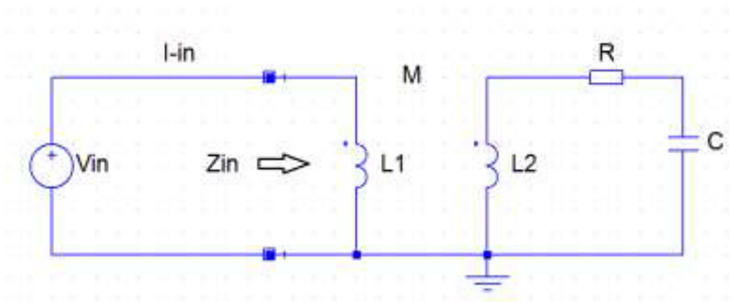

Fig. 1 Equivalent lumped circuit.

3.0 Critical Coupling

The term k^2Q_0 is a ***coupling parameter*** which is varied by adjusting the orientation of the coupling loop *in situ* in order to achieve a match to Z_0 at the normalized working, or coupled resonant frequency, ψ_{match}. The value of k^2Q_0 at which a match is achieved is termed ***critical coupling***. The value of k^2Q_0 required to achieve critical coupling is a function of the loop's normalized self-reactance, x_{L1}. To evaluate k^2Q_0 at critical coupling, force a match by setting $z_{in} = jx_{L1} + (k^2Q_0\, x_{L1}) / (1 + j\psi_{match}) = 1$, then multiply and collect terms:

$$j(\psi_{match} - x_{L1}) + (1 + \psi_{match}\ x_{L1}) = k^2Q_0\, x_{L1} \quad (3)$$

Equation (3) is satisfied only if the imaginary component $j(\psi_{match} - x_{L1})$ equals zero. Thus, ψ_{match} is numerically equal to x_{L1}, further implying that, for critical coupling:

$$k^2Q_0\,(\text{crit}) = (1 + (x_{L1})^2\,) / x_{L1} \quad (4)$$

Figure 2 (below) shows that a singular solution for critical coupling is obtained when $x_{L1} = 1.0$ (50 ohm loop) and $K^2Q_0 = 2$. For higher values of coupling parameter, solutions for a smaller loop or a larger loop are possible, but larger loops may be physically unwieldy and potentially lossy, while it may be difficult to obtain sufficient coupling with a smaller loop. The sweet spot is between 20 and 50 ohms. Please note that **K** in this context should not be confused with κ (Greek letter kappa) as used in (Kajfez, 1999, 2011).

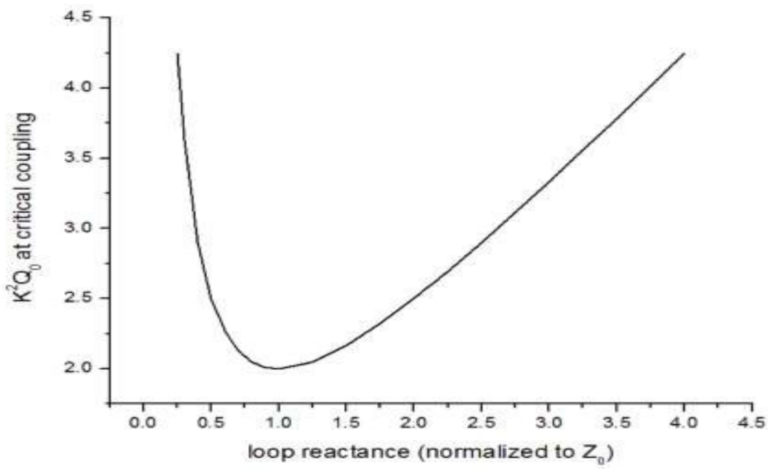

Fig. 2. Coupling parameter versus loop reactance

4.0 Power in the Resonator

When both the transmitter's reverse-output and the resonator's input are matched to Z_0, the frequency for best match coincides with the frequency for maximum power (i.e., maximum dee voltage) in the resonator. However, half the power will then be dissipated in the transmitter's output resistance– an inefficient and expensive solution. More power is available if the transmitter's reverse-output is <u>not</u> matched to Z_0, but then the frequency for best match (minimum SWR) may not coincide with the frequency for maximum power in the resonator.

When the loop is driven directly at its input terminal by an ideal voltage source, power in the resonator is proportional to Re (y_{in}); when driven by an ideal current source, power is proportional to Re (z_{in}). The frequencies for peak power are most conveniently found by plotting Re{z_{in} } and Re{1 / z_{in} } as a function of ψ using Excel^{tm} or equivalent program capable of handling complex arithmetic. For our particular example (50 ohm loop, $k^2Q_0 = 2$) The peak power is at ψ= 2 for the voltage source and ψ= 0 for the current source, while the frequency for best match (minimum standing ratio), is at ψ_{match}= 1. The bandwidth(s) for the ideal voltage and current source are half that of the matched source (2 versus 4 ψ units), indicating a doubling of working or loaded Q.

For the ideal current or voltage sources the frequency for best match can be aligned with the frequency for maximum power in the resonator by driving the coupling loop through an appropriate length of transmission line:

$$\text{Length} = \varphi_{loop} \text{ (in electrical degrees)} = \arctan(1 / x_{L1}) \text{ or, per trig identity} \quad (5)$$

$$\varphi_{loop} = 90^o - \arctan(x_{L1}) \quad (5a)$$

plus an even number (0, 2, 4...) of quarter waves for a low-resistance source, as required to make the physical connection, or an odd number (1, 3, 5...) of quarter waves for a high-resistance source. For the loop in our working example (x_{L1}= 1), the correct length is arctan (1/1)= 45^o for a low-resistance source, or 135^o for a high-resistance source (plus the corresponding number of 1/4 waves needed to make the physical connection). An alternative means of aligning best match with maximum power is described in Appendix B.

5.0 The Q Circle

Deeper insight is gained by transforming the resonator input impedance z_{in} to the complex reflection coefficient $\Gamma = (z_{in} -1) / (z_{in} + 1)$. For $x_{L1} = 1.0$ and K^2Q_0 as parameter, we can evaluate Γ as a function of normalized frequency variable ψ using Excel™ or equivalent.

The result may be plotted on a Cartesian (x, y) grid which is then overlaid on a Smith Chart– a template familiar to RF engineers. Figure 4 shows the locus of Γ for the 50 ohm loop ($x_{L1} = 1.0$) for three values of coupling parameter k^2Q_0 = 0.1, 2.0, and 4.0. When K^2Q_0 is very small, i.e., when the loop is oriented such that minimal magnetic flux links the resonator dee stem, little or no energy is coupled into the resonator proper, and $\Gamma[\psi]$ is essentially that of the isolated loop.

While varying ψ, Γ starts at a point on the perimeter of the Smith Chart corresponding to jx_{L1} = j1, then moves clockwise in a tight circle. When the coupling parameter is increased, the locus of Γ swells to form a larger circle (a Q circle) whose diameter is directed toward the center of the Smith Chart. In this example, when k^2Q_0 is less than or greater than 2, the condition is called 'under-coupled' or 'over-coupled' respectively. If k^2Q_0 = 2.0, the Q circle intersects the center of the Smith Chart at the point $|\Gamma| = 0$ at normalized frequency ψ = 1.0, where z_{in} = 1. This is called critical coupling (as we found earlier) and is the condition where one would preferably operate the RF system.

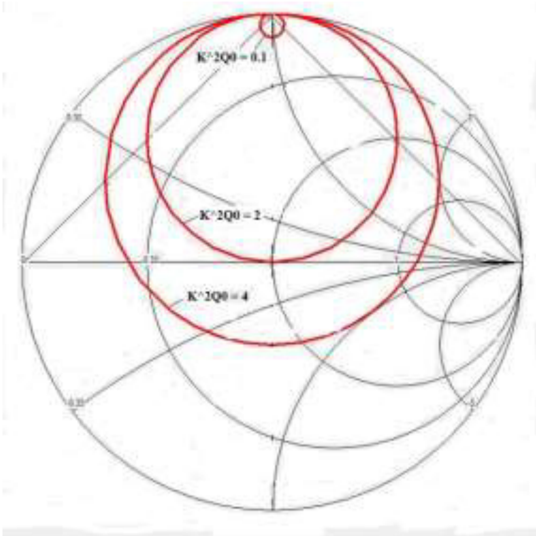

Fig. 4. Q Circles for Various Coupling Parameters

6.0 Best match with maximum power

We found earlier that, for the 50 ohm loop driven by a voltage generator, we could align the frequency for best match with that for maximum power in the resonator by driving the loop through a 45^o length of transmission line. The rationale for this relationship is clearly revealed using the Q circle construction, plotting Γ as a function of frequency, as measured at the transmitter-end of the feed-line. Figure 5 shows two Q circles (in **bold**); one measured directly at the loop (tangent at 90^o on the Chart) and another 45^o electrical degrees upstream from the loop (90^o clockwise, *toward the generator*) per Smith chart convention, to a new tangent position at zero chart degrees.

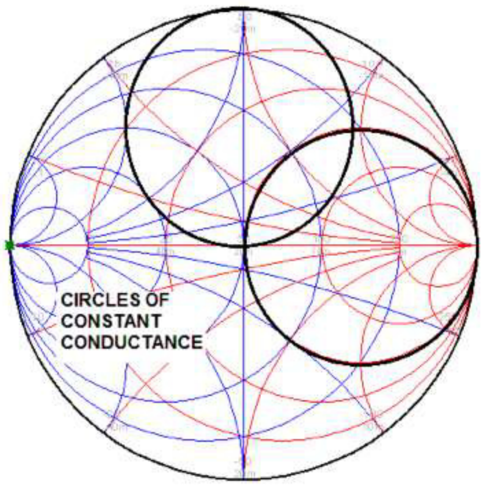

Fig. 5. Voltage source at 0^o and 45^o transmission line length

For the low- resistance (voltage) source as generator, power in the resonator is proportional to input conductance, g, shown as iso-contours which are tangent at the left-most edge of the Smith Chart. These iso-contours describe a topographic map of the *conductance terrain* whose elevation profile is symmetric with respect to the horizontal (i.e., pure resistance) axis of the Smith Chart. Tracing the path of the Q circle starting at zero degrees on the x-axis of the chart, we see that, for a voltage source as generator, a plot of power (gV^2) versus frequency ψ must be symmetric with respect to the horizontal axis and, in particular, must also be symmetric relative to the point of ideal match at $|\Gamma| = 0$. Based on the Q circle construction, another expression for the correct length of feed-line becomes obvious by inspection:

$$\text{Length} = \varphi_{loop} \text{ (in electrical degrees)} = \arg(\Gamma_{loop}) / 2 \qquad (6)$$

where Γ_{loop} is the reflection coefficient of the ***uncoupled loop***, and arg (Γ_{loop}) is the corresponding argument or polar angle expressed in chart degrees. Feed-line lengths for the loop are expressed in electrical degrees measured clockwise toward the generator, and there are 2 chart degrees per electrical degree. In our working example, (Γ_{loop})= $1.0\ _{angle}90^{o}{}_{chart}$, so that $\arg(\Gamma_{loop}) / 2 = 45^{o}{}_{elec}$ plus an even number (0, 2, 4 ...) of 1/4 waves for a low-resistance source, etc.

7.0 A 'degenerate condition'

What happens to the resonator's bandwidth when a high-resistance generator (current source) drives the loop through a 45° feed-line, or a low-resistance generator (voltage source) drives the loop through 135° feed-line? Figure 6 illustrates the former case. The condition of best Z_0 match is independent of source resistance, so the SWR (standing wave ratio) trace is normal. But the power trace is almost flat over the entire 200 kHz frequency sweep, as if system Q has collapsed!

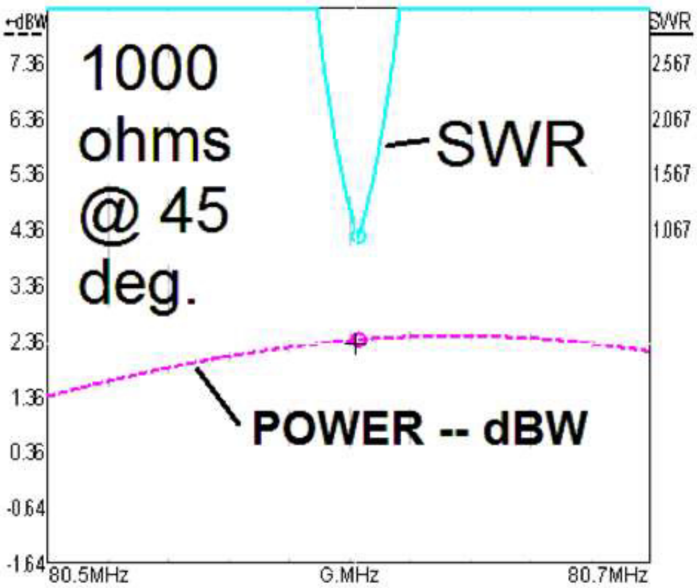

Fig. 6. A degenerate condition

Referring back to Figure 5, the reason is clear; the Q circle, now tangent at zero degrees on the Smith Chart, is superimposed on a circle of constant resistance so that, when driven by a high resistance generator (current source) the power (I^2R) in the resonator– proportional to resistance– is virtually constant over the entire frequency sweep. An analogous condition holds when a voltage source drives the resonator through 135^o and the Q circle is rotated to 180^o on the Chart, where it is superimposed on a circle of constant conductance. Transmission line-length, φ, should be chosen such that the Q circle for a low-resistance generator is placed on the right side of the Smith Chart and, likewise, the Q circle for a high-resistance generator should be positioned on the left side of the Smith Chart.

8.0 Load Faults

While power in the resonator may be constant over a broad frequency sweep in degenerate mode, the same is not true for reactive current or voltage. A high-power transmitter in this mode may be subject to overload and possible damage when operated away from resonance, as typically happens when recovering from load faults, or when hunting for correct resonance (best match) during initial cold start. In any cyclotron RF system, dee sparks are a relatively common occurrence. A well-designed system will incorporate layers of protection, including fast-acting reflected-power sensors, power-supply overload relays, fast-acting circuit-breakers, etc., but an intense dee spark may occasionally propagate back up the feed-line into the anode compartment of the transmitter, potentially causing a damaging air-spark. Our model simulation (see Appendix C) demonstrates that this condition is made much worse in 'degenerate' mode. The transmitter delivering 10 kW to the resonator is modelled as a tetrode (constant-current source) with its tuned output / coupling network.

A load-fault (dee spark) is simulated by shorting the dee to ground with a low-value resistor (box F in the model). With no short, the voltage at the anode is 5.76 kV. Applying the short causes the impedance at the input terminals of the loop to change from 50 ohms resistive to 50 ohms reactive (the loop's reactance). This reactance is transformed upstream through the feed-line, in degenerate mode relative to the low output impedance (3 + j6.5 ohms) of the transmitter. The transmitter's output matching network acts in a manner similar to an additional 90^o section of feed-line, further transforming the impedance and causing the load resistance seen by the anode to jump from 2.4 K ohms to 15.2 K ohms, with a corresponding increase in voltage from 5.76 kV to 37.2 kV. The scale of the increase is determined by the Q's of the various inductors -- deliberately set high by design.

Repeating the simulation, this time with the correct (38^o) non-degenerate length of transmission line, we have 5.74 kV at the anode with no short, and 128 Volts at the anode with the short -- a substantial reduction! However, it should be acknowledged that this analysis is linear and steady-state, rather than a more explicit, but difficult to implement, travelling-wave transient analysis, so that the above result should be regarded as qualitative.

9.0 Complex Source impedance

When the transmitter's output impedance is not purely resistive, the solution requires a feed-line length such that both the Q circle for the resonator and that for the transmitter be aligned with the horizontal axis of the Smith Chart, in turn requiring knowledge of the transmitter's output impedance in order to implement the correct length. The transmitter's normalized output *resistance* can be estimated by means of a measurement, under power, of normalized bandwidth, provided the loop's Q circle is already aligned with the axis of the Smith Chart. Note that bandwidth and frequency-offset measurements under power require temporary disabling of any closed-loop control of frequency. The transmitter's normalized output *resistance*, $R / Z_0 = r_{out}$, varies linearly with the normalized working or coupled bandwidth: $r_{out} = [(F_{high\ 3dB} - F_{low\ 3\ dB}) / (F_0/Q_0)] - 1$. Thus, $r_{out} = 0$ if the normalized bandwidth = 1. When r_{out} is small, $Arg(\Gamma_{gen})$ closely fits the function $Arg(\Gamma_{gen}) = 2\arctan(\Delta)$ i.e., the (r = 0) trace in Figure 7. Analysis by simulation shows that the transmitter's normalized output *reactance* is also approximately equal (within ~5%) to the normalized frequency offset Δ.

Figure 7 is a plot of $Arg(\Gamma_{gen})$ versus normalized frequency offset, incorporating numerous values of generator impedance, loop reactance, etc., but always at critical coupling so as to achieve a Z_0 match at frequency F_{match}. The model is tested within the RF system's working 3 dB bandwidth, using one or more half-wavelengths of feed-line connected to a pre-aligned resonator loop per equation (5A) or (6).

$$\varphi_{gen} = \arg(\Gamma_{gen}) / 2 + 90^{o} \quad \text{– electrical degrees} \tag{7}$$

The extra 90^{o} is needed to place the net Q circle on the non-degenerate side of the Smith Chart. The length of feed-line required to align both resonator and generator is the sum of $\varphi_{loop} + \varphi_{gen}$. Combining equations (6) and (7);

$$\varphi_{total} = \varphi_{loop} + \varphi_{gen} = 90^{o} + \mathrm{Arg}(\Gamma_{loop}) / 2 + \mathrm{Arg}(\Gamma_{gen}) / 2 \tag{8}$$

Arg(Γ_{gen}) is measured under normal operating power conditions, starting with a test-line of total length φ_{loop} plus as many 180^{o} lengths as required to connect the transmitter to the resonator, plus at least one additional 180^{o} length to allow trimming (instead of adding more line) to achieve the correct final length. After trimming, both the resonator's and transmitter's Q circles are now presumed to be aligned with the horizontal axis of the Smith Chart, but unless the transmitter's output impedance turns out to be purely resistive, the composite Q circle, measured at the transmitter end of the feed line, will not be aligned with the horizontal axis of the Smith Chart.

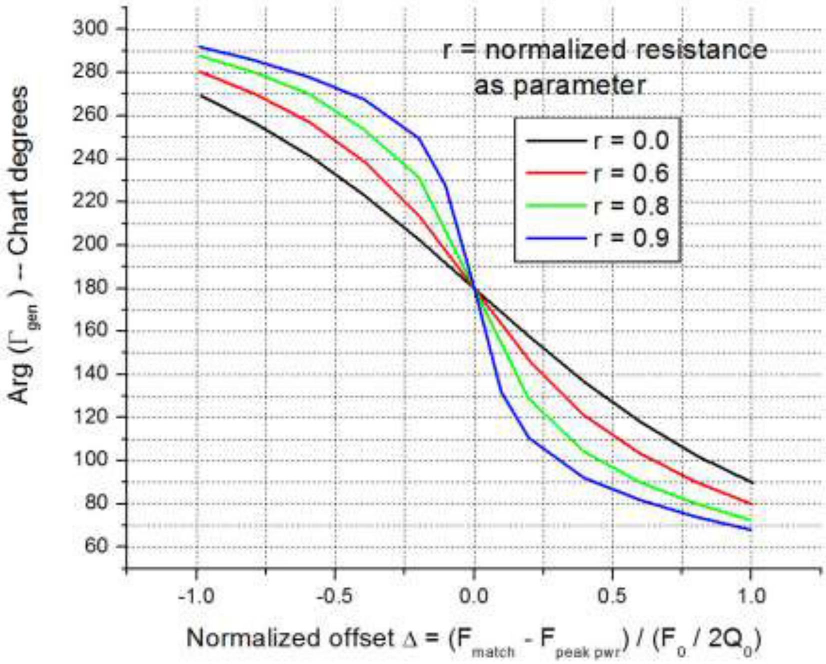

Fig. 7. Arg (Γ_{gen})

10. Summary and Conclusion

Algebraic manipulation and normalization of the network variables describing the resonator's input impedance yields a compact and general formula, providing useful insight into inter-relationships between various network parameters. Moving from general to specific system design is greatly facilitated by modelling a resonator's hardware implementation using RF simulation software such as ***SimSmith*** or equivalent using values ω_0 (or F_0), Q_0, X_{L1}, etc., measured 'cold', *in situ*, using an instrument such as a vector network analyzer in combination with techniques and procedures outlined in [Kajfez, 1999, 2011]. Maximum RF power transfer from transmitter to the high-Q cyclotron resonator is obtained when the transmitter's output resistance is either very low (a voltage source) or very high (a current source) and not matched to the transmission line's characteristic impedance Z_0. In order to align the frequency for best Z_0 match at the load with the frequency for maximum power in the resonator, the Q circle representing $\Gamma(z_{in})$ at the loop's input terminal can be transported upstream toward the transmitter by an appropriate length of feed-line, or the coupling loop's self inductance can be tuned out with a series capacitor (per Appendix B), In either case, the loop's Q circle axis becomes aligned with the real (resistance) axis of the Smith Chart aligning, in turn, best match with max power. If the transmitter's output impedance is complex (resistance plus reactance) an additional compensating length of feed-line is required to align the transmitter's Q circle. Output resistance and reactance and the corresponding length of additional feed-line are revealed by measurement (under power) of normalized bandwidth and normalized frequency offset Δ, respectively. A wide-band 'degenerate' operating mode is explained and analyzed; one is cautioned to avoid this mode due to risk of overload and potential damage to equipment.

Appendix A: Derivation of eq. (2)

Start with Eq.(1): $Z_{in} = j\omega L_1 + \{\omega^2 M^2\} / \{j[\omega L_2 - 1/\omega C] + R\}$.
Substitute: $M^2 = k^2 L_1 L_2$; $C = 1 / (Q_0\omega_0 R)$; $L_2 = (Q_0 R/\omega_0)$;
to yield $Z_{in} = j\omega L_1 + [k^2 Q_0 \omega L_1 R(\omega / \omega_0)] / [jQ_0 R[(\omega / \omega_0) - (\omega_0 / \omega)] + R]$
The R's all cancel; strike out the (ω / ω_0) term in the numerator which is ~1 near resonance, assuming Q_0 is in the thousands. Normalize the loop reactance term with respect to Z_0; $\omega L_1 = x_{L1}$;
Substitute: $\psi = Q_0 [(\omega / \omega_0) - (\omega_0 / \omega)]$.
$= Q_0 [\omega^2 - \omega_0^2] / \omega\omega_0$
$= Q_0 (\omega - \omega_0) (\omega + \omega_0) / \omega\omega_0$
Near resonance, $\omega \sim \omega_0$, assuming Q_0 is in the thousands, so that,
$\psi = 2Q_0 (\omega - \omega_0) / (\omega_0)$, or
$\psi = (\omega - \omega_0) / (\omega_0 / 2Q_0)$.
Thus, ψ = frequency deviation $(\omega - \omega_0)$ normalized with respect to the un-coupled resonator's half bandwidth, and:
$z_{in} = jx_{L1} + (k^2 Q_0 x_{L1}) / (1 + j\psi)$

Appendix B: An alternative means of aligning best match with maximum power

The method entails 'tuning out' the self-inductance of the coupling loop with a series capacitor placed directly at the input to the loop, assuming L and C reactances effectively cancel over the frequency range of interest (true for a high-Q resonator), further resulting in a modified expression for z_{in} in which the leading term, jx_{L1} vanishes, yielding $z_{in} = (k^2 Q_0 x_{L1}) / (1 + j\psi)$. To determine Critical Coupling, set $z_{in} = 1$, as before, so that $(k^2 Q_0 x_{L1}) = (1 + j\psi)$. This is satisfied only if $\psi = 0$ (i.e., the transmission line is matched at ω_0 - the resonant frequency of the isolated, un-coupled resonator). Thus, for this example $(k^2 Q_0)_{crit} = 1/ x_{L1}$. Note that this scheme may be of more theoretical, rather than practical interest since, in our experience, a compact high-current RF capacitor may be a relatively 'weak link' reliability-wise. Moreover, when driven by a transmitter with a low output impedance (a voltage source) the resulting Q-circle ends up situated on the 'degenerate' side of the Smith Chart, necessitating addition (or subtraction) of a 90^o section of transmission line between transmitter and loop.

Appendix C: SimSmith resonator model simulation

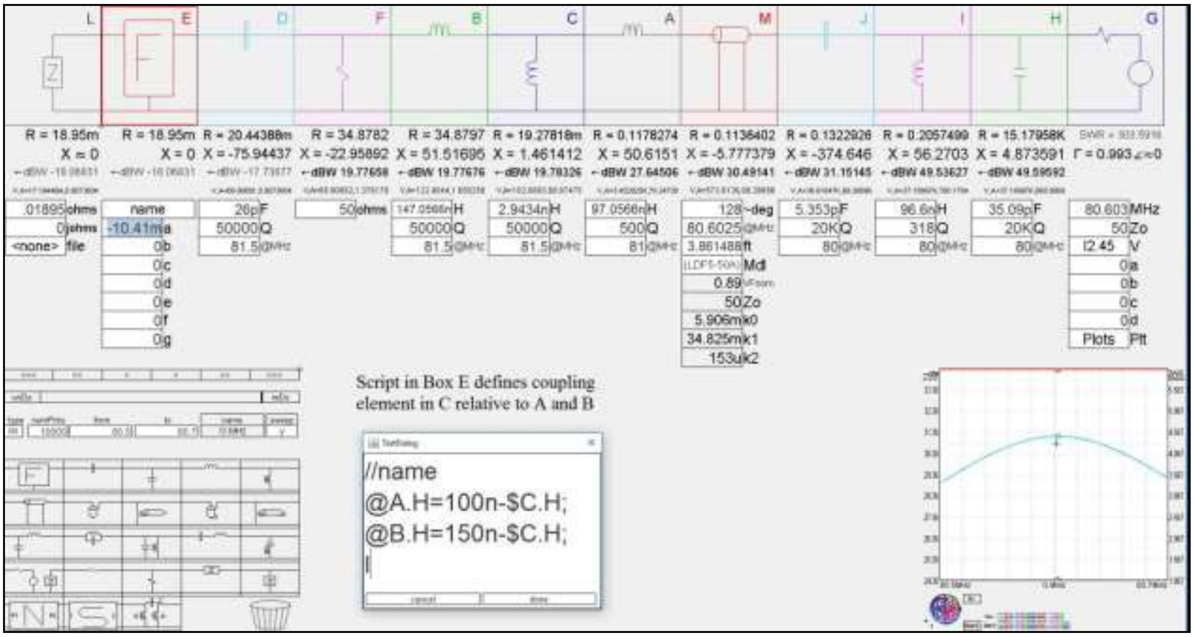

Fig. C1 Simulation Screen Shot

The transmitter is at the right side of the diagram, modelled as a tetrode current source G with anode capacitance H, and matching network elements I and J. Note that the SimSmith model does not use magnetic coupling *per se*, but rather employs a tee-equivalent circuit, including a script embedded in Box E, so that the mutual inductance element in C can be precisely dialled in to establish critical coupling, while simultaneously varying inductors A (loop) and B (dee stems) as required by the tee-equivalent formulation. The 128^{o} feed-line in M places the system's Q-circle on the 'degenerate' side of the Smith Chart; the Dee capacitance (Box D) is shorted to ground by the 50 ohm resistor in F to create a fault condition.

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